About the mathematical models of the COVID-19 epidemic

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INTRODUCTION

Mathematical models of the epidemics development always attracted the attention in mathematics and medicines.

Models require:

1) Describe the course of the epidemic by mathematical equations. This is makes possible to provide for the necessary actions when it is repeated;
2) Give a forecast of the development of the current epidemic based on information about its initial period.

The second fact is important. It makes it possible to quickly provide for the required number of doctors, medicines, hospital beds, and thereby reduce the number of sick's and deaths. But the accuracy of this prediction depends on the length of the initial period and the reliability of the statistical data obtained, which often have large errors. Therefore, there are many models, and the creation and research of new ones continues.

Most of the models use a systems of differential equations that take into account the total number of inhabitants *S*, the change over time in the number of sick *I*, recovered and died *R*. These are, for example, *SIR*, *SEIR*, etc.

The disadvantages of these models and others similar to them are:

- the inability to obtain analytical expressions that describe the course of the epidemic;
- the need for numerical solution of systems of differential equations, which complicates the analysis of the results;
- when solving, it is necessary to find the values of several parameters describing the course of the epidemic, and this often leads to ambiguity of the results obtained.

When studying the statistics on the coronavirus epidemic in various countries, an unexpected fact was discovered. The dependence of the number of patients on time is well described by the Gaussian function

$$f(t) = A\exp\left(-\frac{(t-t_0)^2}{2\sigma^2}\right).$$

In our work, this simple mathematical model was tested on data on the coronavirus epidemic in Austria and used to describe the coronavirus epidemic in Ukraine.

MODEL OF THE CORONAVIRUS EPIDEMIC IN AUSTRIA (2020 year)

We used statistical data on the course of the coronavirus epidemic in Austria for the period from February 26 to May 25, 2020 - the development of the epidemic and its decline (the number of active patients in medical terminology). The simulation results are shown in Fig. 1.

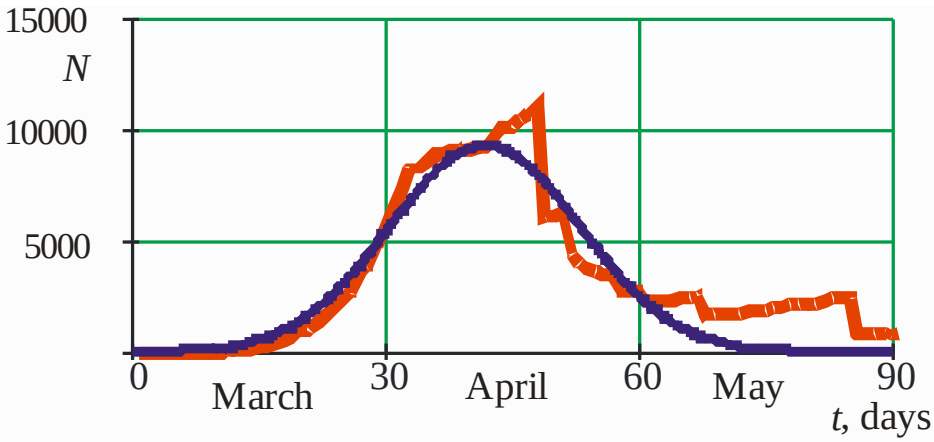

Fig. 1. Coronavirus epidemic in Austria in 2020 year

The thick red line is statistical data, the thin blue line is the Gaussian least squares fit. According to the approximation, the maximum of the epidemic should occur on the 42nd day (April 7). The number of patients on this day is 9626. In fact, the maximum came on the 48th day (April 13), the number of patients on that day was 11,101. This is a good agreement between statistical and calculated data. The downward curve does not describe the course of the epidemic as well as the upward curve. The reason for this is the jumps on the statistical curve, where the real situation is shown inaccurately.

A numerical experiment was carried out to predict epidemic progress: based on data for the first 30 days. The results are shown in Fig. 2.

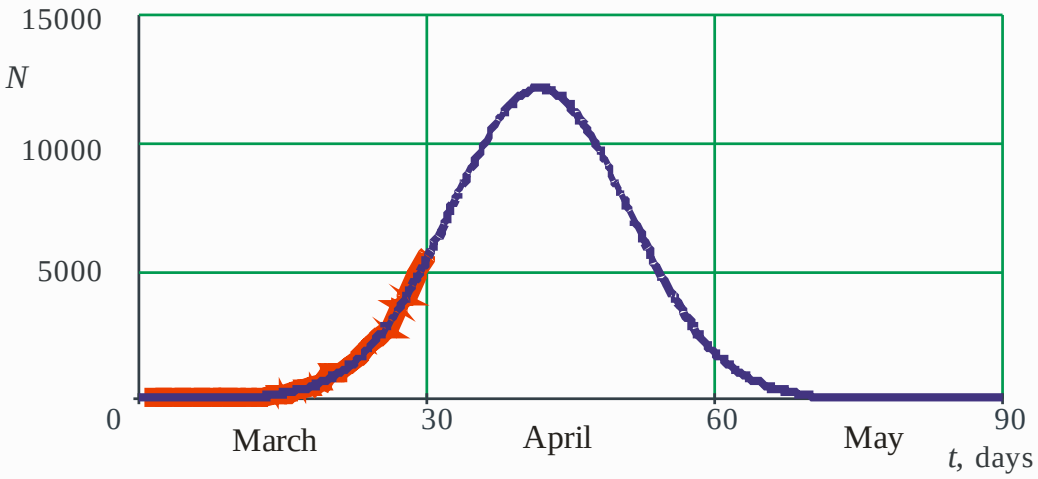

Fig. 2. Forecast of the development of the coronavirus epidemic in Austria in 2020

According to the forecast the maximum will come on the 42nd day. The number of patients on this day is 12097. This fits coincides well with the statistics.

MODEL OF THE CORONAVIRUS EPIDEMIC IN UKRAINE

The results of using a mathematical model based on the Gaussian function in Ukraine for the period from March 4 to May 25, 2020 (the first wave of the epidemic) and the forecast for the future are shown in Fig. 3.

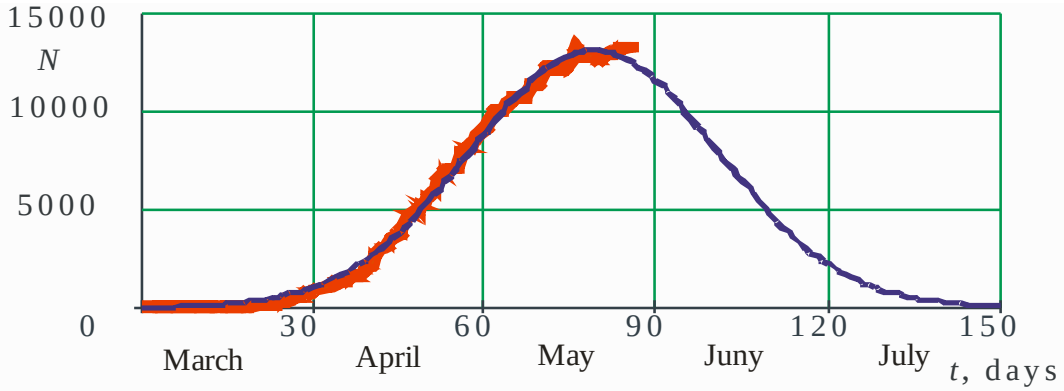

Fig. 3. Coronavirus epidemic in Ukraine in 2020 (first wave)

The development of the epidemic in Ukraine was much slower than in Austria. There the maximum on the chart was on the 48th day (April 13), in Ukraine - much later (according to the forecast - on the 78th day, May 19). This was consistent with medical predictions. The number of patients should have dropped significantly by the end of June. But at this time, the second wave of coronavirus began, followed by the third (Fig. 4), and the number of patients did not decrease, but, starting in September, began to increase. The number of patients in the second and third waves exceeded 400 thousand. This is much more than in the first wave, where it did not reach 15 thousand.

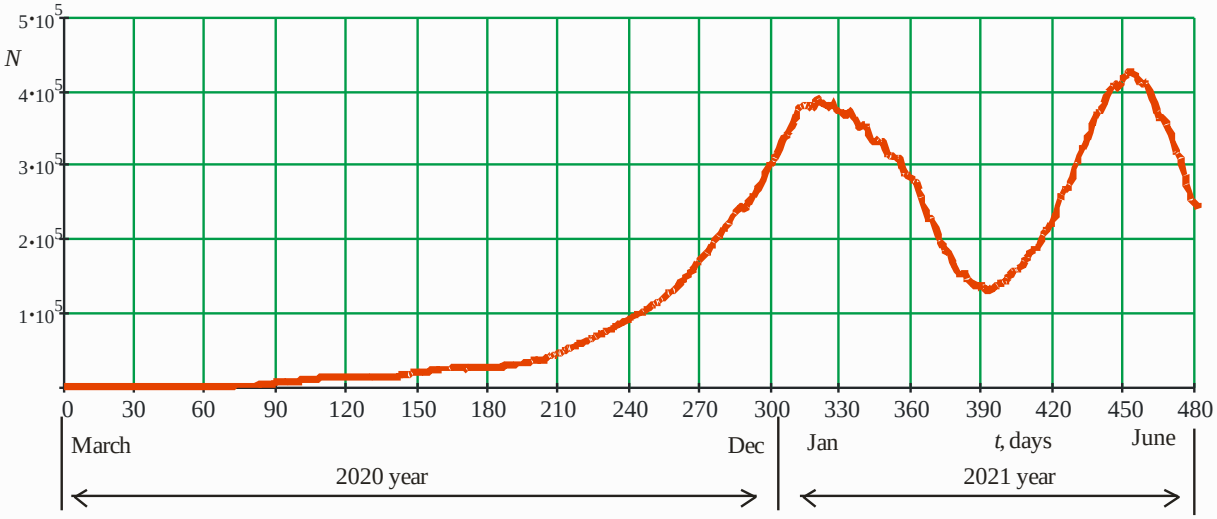

Fig. 4. Coronavirus epidemic in Ukraine in 2020-2021 years

The mathematical model used does not take into account the onset of the next wave of coronavirus, and therefore the forecast of the course of the epidemic shown in Fig. 3, did not come true. But the model can be used to analyze and forecast each wave of the epidemic separately.

In fig. 5 and 6 show the application of a mathematical model to analyze the course of the epidemic in Ukraine in 2020 and 2021 years during the second and third waves. The model describes each period of the epidemic well.

Fig. 6 it can be assumed that the epidemic in Ukraine will end in August-September 2021.year. A necessary condition is that the next wave will not appear in the fall.

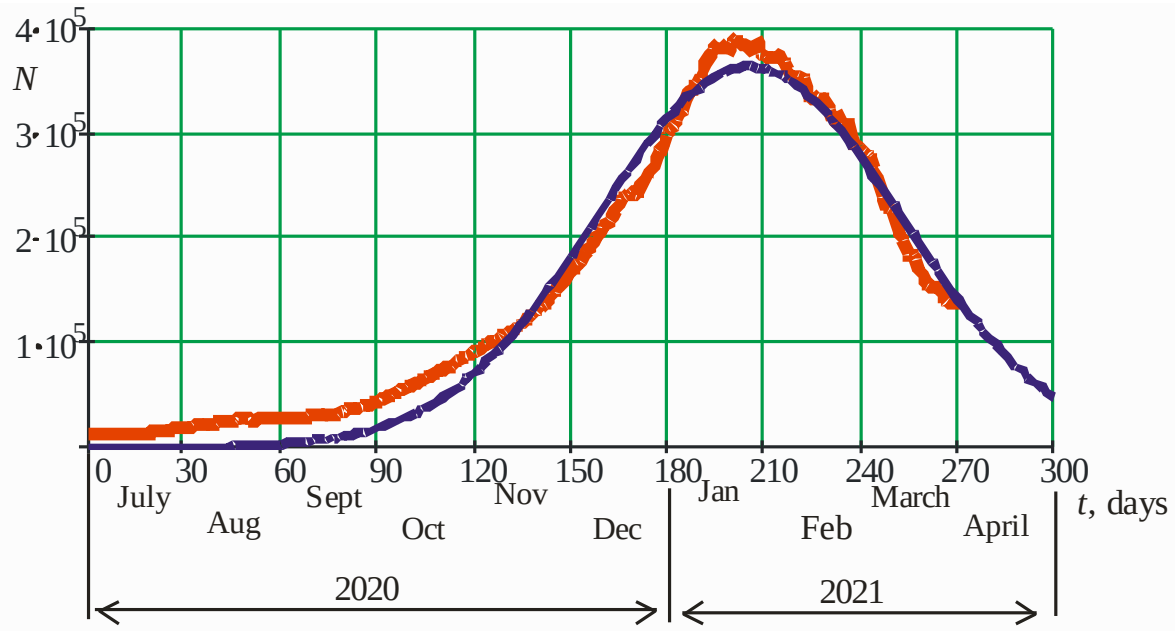

Fig. 5. Coronavirus epidemic in Ukraine in 2020-2021 (second wave)

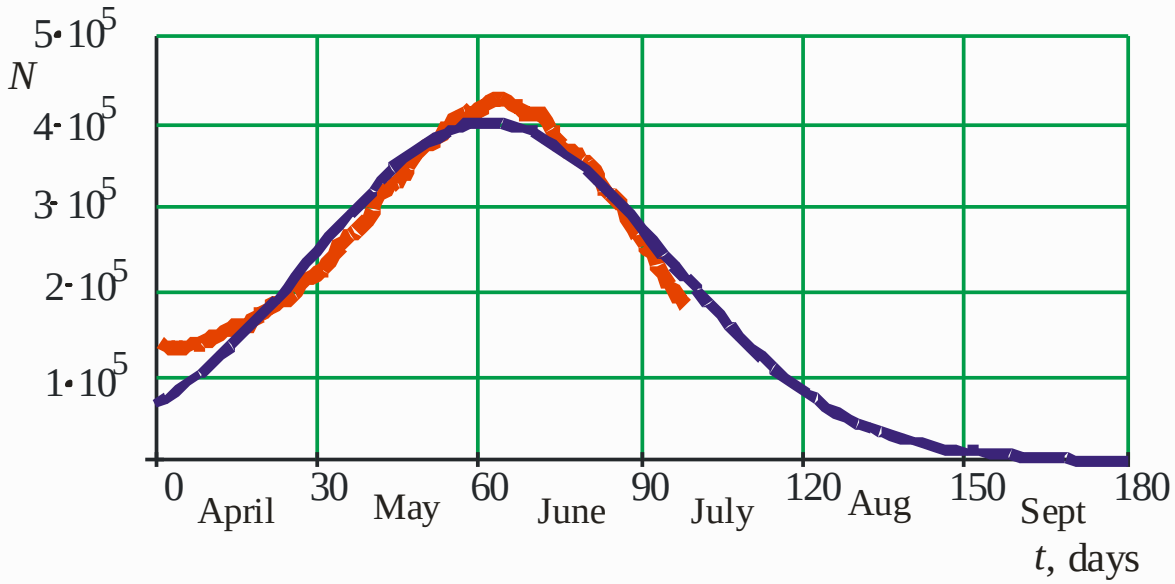

Fig. 6. Coronavirus epidemic in Ukraine in 2021 (third wave)

CONCLUSION

1. A simple mathematical model describing the course of the coronavirus epidemic has been proposed and investigated. The model is based on Gaussian approximation.
2. The model describes well the course of the epidemic and allows predicting its development in the future based on information about the initial period.
3. Analysis of the course of the epidemic makes it possible to determine the number of cases, the time of maximum and the time of exit from the epidemic.
4. The disadvantage of the model is the inability to take into account the next wave of the epidemic.